NONISOTHERMAL GENERALIZED COUETTE FLOW OF A FLUID WITH A POWER-LAW RHEOLOGICAL CHARACTERISTIC

S. A. Bostandzhiyan and V. I. Boyarenko UDC 532.543:532.135

The nonisothermal steady flow of a power-law fluid between two parallel plates is analyzed for different kinds of temperature boundary conditions and, moreover, with energy dissipation taken into account. It is assumed that the fluidity of the substance is a linear function of the temperature.

The nonisothermal generalized Couette flow of a Newtonian fluid was studied in [1]. The viscosity of the fluid was assumed to vary with the temperature hyperbolically, i.e., its fluidity was assumed to vary with the temperature linearly. In the practical case of a molten or dissolved polymer flowing through the helical channel of a screw pump or extruder it is of interest to analyze the analogous problem involving non-Newtonian fluids. As is well known, the rheological characteristics of many such substances are represented by a power law. In this article we will consider the flow of a power-law fluid between two parallel plates, with one plate moving at a constant velocity and with a pressure gradient in the separating gap. The temperature is constrained by either of two sets of boundary conditions: a) both plates are maintained at constant and, in the general case, different temperatures (housing and screw are both thermostaticized), or b) the upper plate is thermostaticized while the thermal flux at the lower plate is zero (the screw is thermally insulated). The relation between fluidity and temperature is assumed linear.

We consider the motion of a fluid between two parallel infinitely large plates $y = 0$ and $y = h$. The upper plate is moving at a constant velocity v_0 along the x-axis while the lower plate remains stationary. In the gap between the plates there exists a constant pressure gradient $dp/dx = A > 0$. The plate temperature T_1 and T_2 are given.

The rheological power law will be defined as

$$\frac{dv}{dy} = k\,|\tau|^{n-1}\,\tau. \tag{1}$$

It is assumed that coefficient k, which characterizes the fluidity of the substance, is a linear function of the temperature, i.e.,

$$k = k_0\,[1 + \beta\,(T - T_2)] \qquad (k_0,\ \beta = \text{const}). \tag{2}$$

The flow and the heat transfer, with energy dissipation taken into account, are represented by the following system of equations

$$\frac{d\tau}{dy} = A, \qquad \frac{d^2T}{dy^2} + \frac{\tau}{\lambda J}\cdot\frac{dv}{dy} = 0. \tag{3}$$

This system is to be solved for the following boundary conditions:

$$v = 0,\quad T = T_1 \quad \text{at} \quad y = 0; \quad v = v_0,\quad T = T_2 \quad \text{at} \quad y = h. \tag{4}$$

The first equation yields

$$\tau = A\,(y - y_0), \tag{5}$$

with the integration constant y_0 being the ordinate of the plane of zero shearing stress.

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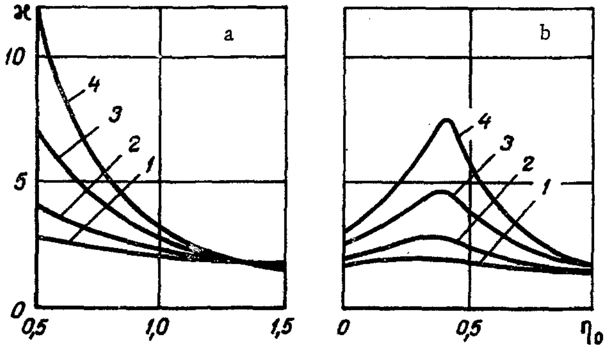

Fig. 1. Curves of $\varkappa$ versus η_0 for various values of n: 1) n = 0; 2) 1; 3) 3; 4) 5.

With (1) and (5), system (3) and the boundary conditions can be written in the dimensionless form:

$$\frac{dw}{d\eta} = \frac{1}{\alpha} |\eta - \eta_0|^{n-1} (\eta - \eta_0)\,\theta, \quad \frac{d^2\theta}{d\eta^2} + \varkappa^4 |\eta - \eta_0|^{n+1}\theta = 0; \tag{6}$$

$$w = 0, \quad \theta = \theta_0 \quad \text{at} \quad \eta = 0; \quad w = 1, \quad \theta = 1 \quad \text{at} \quad \eta = 1. \tag{7}$$

The solution to the second of Eqs. (6) is expressed in terms of Bessel functions [2] differing in the sign of the variable $(\eta - \eta_0)$:

$$\begin{gathered}\theta_1 = \sqrt{\eta - \eta_0}\left\{A_1' J_{1/\nu}\left[\frac{2\varkappa^2}{\nu}(\eta - \eta_0)^{\frac{\nu}{2}}\right] + B_1' J_{-1/\nu}\left[\frac{2\varkappa^2}{\nu}(\eta - \eta_0)^{\frac{\nu}{2}}\right]\right\}, \\ (\eta \geqslant \eta_0), \\ \theta_2 = \sqrt{\eta_0 - \eta}\left\{A_2' J_{1/\nu}\left[\frac{2\varkappa^2}{\nu}(\eta_0 - \eta)^{\frac{\nu}{2}}\right] + B_2' J_{-1/\nu}\left[\frac{2\varkappa^2}{\nu}(\eta_0 - \eta)^{\frac{\nu}{2}}\right]\right\}, \\ (\eta \leqslant \eta_0),\end{gathered} \tag{8}$$

with the integration constants A_1', A_2', B_1', B_2' and with $\nu = n + 3$. With the Bessel functions represented as power series [2], we have

$$\theta_1 = A_1 F_1(\eta) + B_1 G_1(\eta), \quad \theta_2 = A_2 F_2(\eta) + B_2 G_2(\eta). \tag{9}$$

Here A_1, A_2, B_1, B_2 are new integration constants and

$$\begin{gathered}F_1(\eta) = \sum_{k=0}^{\infty} a_k (\eta - \eta_0)^{\nu k + 1}, \quad G_1(\eta) = \sum_{k=0}^{\infty} b_k (\eta - \eta_0)^{\nu k}, \\ F_2(\eta) = \sum_{k=0}^{\infty} a_k (\eta_0 - \eta)^{\nu k + 1}, \quad G_2(\eta) = \sum_{k=0}^{\infty} b_k (\eta_0 - \eta)^{\nu k}, \\ a_k = \frac{(-1)^k}{k!} \cdot \frac{\varkappa^{4k}}{\nu^k (1+\nu)(1+2\nu)\ldots(1+k\nu)}, \\ b_k = \frac{(-1)^k}{k!} \cdot \frac{\varkappa^{4k}}{\nu^k (\nu - 1)(2\nu - 1)\ldots(k\nu - 1)}.\end{gathered} \tag{10}$$

Depending on the values of parameters $\varkappa$ and α, we can encounter different situations: $\eta_0 \le 0$, $0 \le \eta_0 \le 1$, $\eta_0 \ge 1$. In each of these three cases the formulas for the temperatures, the velocities, and the flow rate will be different. We will consider here all three cases and will determine the ranges of parameter $\varkappa$ and α values within which the different flow modes occur.

Let $0 \le \eta_0 \le 1$. Then, from the condition of equal temperatures and of equal derivatives of the temperature at $\eta = \eta_0$ follows $A_2 = -A_1$ and $B_2 = B_1$. Thus,

$$\theta_1 = A_1 F_1(\eta) + B_1 G_1(\eta), \ (\eta \geqslant \eta_0); \ \theta_2 = -A_1 F_2(\eta) + B_1 G_2(\eta), \ (\eta \leqslant \eta_0). \tag{11}$$

Satisfying the boundary conditions (7) yields

$$\begin{gathered}A_1 = \frac{1}{\Delta}[G_2(0) - G_1(1)\theta_0], \quad B_1 = \frac{1}{\Delta}[F_1(1)\theta_0 + F_2(0)], \\ \Delta = F_1(1) G_2(0) + F_2(0) G_1(1).\end{gathered} \tag{12}$$

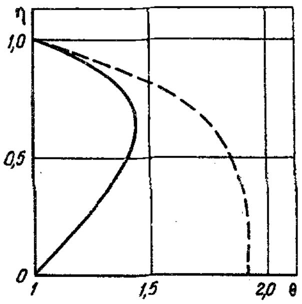

Fig. 2

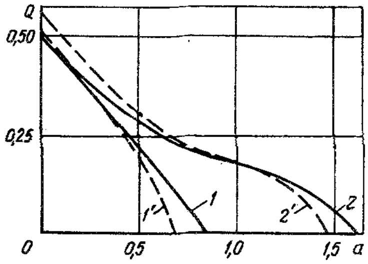

Fig. 3

Fig. 2. Temperature profile at different temperature boundary conditions.

Fig. 3. Curves of flow rate as a function of the pressure gradient, at various values of n: 1, 1') n = 1; 2, 2') n = 3.

Integrating the first of Eqs. (6) with (11) and boundary conditions (7) taken into consideration, we obtain the velocity profile

$$w = 1 - \frac{1}{\alpha}\,[A_1 f_1(\eta) + B_1 g_1(\eta)], \quad (\eta \geqslant \eta_0),$$

$$w = \frac{1}{\alpha}\,[A_1 f_2(\eta) - B_1 g_2(\eta)], \quad (\eta \leqslant \eta_0);$$

$$f_1(\eta) = \sum_{k=0}^{\infty} \frac{a_k}{\nu k + n + 2}\,[(1-\eta_0)^{\nu k+n+2} - (\eta-\eta_0)^{\nu k+n+2}],$$

$$g_1(\eta) = \sum_{k=0}^{\infty} \frac{b_k}{\nu k + n + 1}\,[(1-\eta_0)^{\nu k+n+1} - (\eta-\eta_0)^{\nu k+n+1}], \tag{13}$$

$$f_2(\eta) = \sum_{k=0}^{\infty} \frac{a_k}{\nu k + n + 2}\,[\eta_0^{\nu k+n+2} - (\eta_0-\eta)^{\nu k+n+2}],$$

$$g_2(\eta) = \sum_{k=0}^{\infty} \frac{b_k}{\nu k + n + 1}\,[\eta_0^{\nu k+n+1} - (\eta_0-\eta)^{\nu k+n+1}].$$

Satisfying the condition of equal velocities at $\eta = \eta_0$ yields an expression for η_0:

$$A_1[f_1(\eta_0) + f_2(\eta_0)] + B_1[g_1(\eta_0) - g_2(\eta_0)] = \alpha. \tag{14}$$

An expression for the dimensionless flow rate will be obtained by integrating (13) with respect to η from 0 to η_0 and from η_0 to 1 and adding the results:

$$Q = 1 - \eta_0 - \frac{1}{\alpha}\left\{A_1 \sum_{k=0}^{\infty} \frac{a_k}{\nu k + n + 3}\,[(1-\eta_0)^{\nu k+n+3} - \eta_0^{\nu k+n+3}]\right.$$

$$\left. + B_1 \sum_{k=0}^{\infty} \frac{b_k}{\nu k + n + 2}\,[(1-\eta_0)^{\nu k+n+2} + \eta_0^{\nu k+n+2}]\right\}. \tag{15}$$

When $\eta_0 \leq 0$, the temperature profile is determined from the first of expressions (11). Having found the integration constants A_1 and B_1 from the boundary conditions at the plates, we obtain

$$A_1 = \frac{1}{\Delta_1}\,[G_1(0) - G_1(1)\,\theta_0], \quad B_1 = \frac{1}{\Delta_1}\,[F_1(1)\,\theta_0 - F_1(0)],$$
$$\Delta_1 = F_1(1)\,G_1(0) - F_1(0)\,G_1(1). \tag{16}$$

The velocity profile is determined from the first of expressions (13), which has been derived from the boundary condition at the moving plate. The boundary condition at the stationary plate is used for determining η_0:

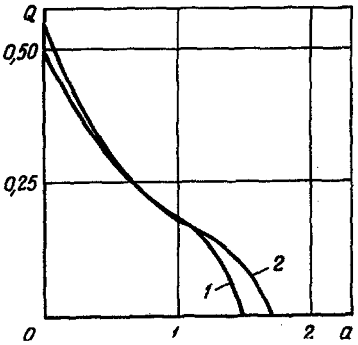

Fig. 4. Effect of parameter $\varkappa_m$ on the $Q(\alpha)$ relation: 1) $\varkappa_m = 1$; 2) 2.52.

$$A_1 f_1(0) + B_1 g_1(0) = \alpha. \tag{17}$$

Integration of the velocity profile from 0 to 1 yields the dimensionless flow rate:

$$Q = 1 - \frac{1}{\alpha}\left(A_1 \sum_{k=0}^{\infty} \frac{a_k}{\nu k + n + 2}\left\{(1-\eta_0)^{\nu k+n+2} - \frac{1}{\nu k + n + 3}\left[(1-\eta_0)^{\nu k+n+3} - (-\eta_0)^{\nu k+n+3}\right]\right\} + B_1 \sum_{k=0}^{\infty} \frac{b_k}{\nu k + n + 1}\left\{(1-\eta_0)^{\nu k+n+1} - \frac{1}{\nu k + n + 2}\left[(1-\eta_0)^{\nu k+n+2} - (-\eta_0)^{\nu k+n+2}\right]\right\}\right). \tag{18}$$

When $\eta_0 \geq 1$, the temperature profile is determined from the second of expressions (11), where

$$A_1 = \frac{1}{\Delta_2}[G_2(0) - G_2(1)\theta_0], \quad B_1 = \frac{1}{\Delta_2}[F_2(0) - F_2(1)\theta_0],$$
$$\Delta_2 = F_2(0)G_2(1) - F_2(1)G_2(0). \tag{19}$$

The second of expressions (13) is used for determining the velocity profile. In integrating the respective differential equation, use was made of the boundary conditions at the stationary plate. Satisfying the boundary condition at the moving plate yields an expression for η_0:

$$A_1 f_2(1) - B_1 g_2(1) = \alpha. \tag{20}$$

The flow rate is

$$Q = \frac{1}{\alpha}\left(A_1 \sum_{k=0}^{\infty} \frac{a_k}{\nu k + n + 2}\left\{\eta_0^{\nu k+n+2} - \frac{1}{\nu k + n + 3}\left[\eta_0^{\nu k+n+3} - (\eta_0 - 1)^{\nu k+n+3}\right]\right\} - B_1 \sum_{k=0}^{\infty} \frac{b_k}{\nu k + n + 1}\left\{\eta_0^{\nu k+n+1} - \frac{1}{\nu k + n + 2}\left[\eta_0^{\nu k+n+2} - (\eta_0 - 1)^{\nu k+n+2}\right]\right\}\right). \tag{21}$$

It is evident from (5) that η_0 characterizes the shearing stress in the gap and at the plates. If both plates were stationary ($\alpha = 0$) then $\eta_0 = 0.5$ with the plates at the same temperatures but $\eta_0 \neq 0.5$ with the plates at different temperatures, within the gap in either case.

It follows from the formulas for A_1 and B_1, also from the expressions for the velocity profile, that $\alpha \to \pm\infty$ as Δ, Δ_1, and Δ_2 approach zero. One may conclude, then, that, as the plate velocity increases, η_0 will increase but remain below some limit defined by the first root of either one of the three equations

$$\Delta(\varkappa, \eta_0) = 0, \quad \Delta_1(\varkappa, \eta_0) = 0, \quad \Delta_2(\varkappa, \eta_0) = 0. \tag{22}$$

In other words, the shearing stresses at the plates when one moves at an infinitely high velocity will tend not toward infinity, as in the isothermal problem [3], but to some finite limit. Since the expressions for Δ, Δ_1, and Δ_2 do not contain θ_0, hence this limit η_0 does not depend on the temperature drop at the boundaries.

It is to be noted that the results in [1] apply to the special case n = 1 and the results with $\varkappa = 0$ are found in [3]. In the case of a pure Couette flow (A = 0), the temperature profile, the velocity profile, and the flow rate are

$$\theta = C_1 \cos(\varkappa^2\eta) + C_2 \sin(\varkappa^2\eta),$$
$$w = \frac{1}{\alpha\varkappa^2}\{C_1 \sin(\varkappa^2\eta) + C_2[1 - \cos(\varkappa^2\eta)]\},$$
$$Q = \frac{1}{\alpha\varkappa^4}\{C_1[1 - \cos(\varkappa^2)] + C_2[\varkappa^2 - \sin(\varkappa^2)]\}.$$

Here

$$C_1 = \theta_0, \quad C_2 = \frac{1 - \theta_0\cos(\varkappa^2)}{\sin(\varkappa^2)}, \quad \alpha = \frac{v_0}{k_0 h C}, \quad \varkappa^4 = \frac{k_0 \beta h^2 C^{1+\frac{1}{n}}}{\lambda J},$$

and C is the root of the transcendental equation

$$C_1 \sin(\varkappa^2) + C_2 [1 - \cos(\varkappa^2)] = \alpha \varkappa^2.$$

Let the given temperatures at the upper plate be constant while the lower plate is thermally insulated. Instead of the former boundary conditions (7) regarding the temperature, we now have

$$\frac{d\theta}{d\eta} = 0 \quad \text{at} \quad \eta = 0. \tag{23}$$

The formulas for determining the temperature profile, the velocity profile, and the flow rate in the gap, also the equations for determining η_0 will all remain unchanged, but the coefficients A_1 and B_1 determined from the new boundary conditions will be

$$A_1 = \frac{G_4(0)}{\Delta}, \quad B_1 = \frac{F_4(0)}{\Delta}, \quad \Delta = F_1(1)\,G_4(0) + F_4(0)\,G_1(1), \quad (0 \leqslant \eta_0 \leqslant 1),$$

$$A_1 = \frac{G_3(0)}{\Delta_1}, \quad B_1 = -\frac{F_3(0)}{\Delta_1}, \quad \Delta_1 = F_1(1)\,G_3(0) - F_3(0)\,G_1(1), \quad (\eta_0 \leqslant 0),$$

$$A_1 = \frac{G_4(0)}{\Delta_2}, \quad B_1 = \frac{F_4(0)}{\Delta_2}, \quad \Delta_2 = F_4(0)\,G_2(1) - F_2(1)\,G_4(0), \quad (\eta_0 \geqslant 1).$$

$$F_3(\eta) = \sum_{k=0}^{\infty} a_k (\nu k + 1)(\eta - \eta_0)^{\nu k}, \quad G_3(\eta) = \sum_{k=0}^{\infty} b_k \nu k (\eta - \eta_0)^{\nu k - 1},$$

$$F_4(\eta) = \sum_{k=0}^{\infty} a_k (\nu k + 1)(\eta_0 - \eta)^{\nu k}, \quad G_4(\eta) = \sum_{k=0}^{\infty} b_k \nu k (\eta_0 - \eta)^{\nu k - 1}.$$

The formulas derived for the case of thermostaticized plates remain valid for a pure Couette flow, only the coefficients C_1 and C_2 change to: $C_1 = 1/\cos(\varkappa^2)$ and $C_2 = 0$.

Numerical calculations according to these formulas have been performed on a computer and the results are shown in Figs. 1-4.

Curves of $\varkappa$ as a function of η_0, based on Eqs. (22), are shown in Fig. 1a for various values of n. As point $(\varkappa, \eta_0)$ approaches such a curve, parameter $\alpha \to -\infty$. From the expressions for Δ, Δ_1, and Δ_2 it appears that, at a fixed value of $\varkappa$, these quantities are functions of η_0 symmetrical with respect to $\eta_0 = 0.5$. Therefore, the points on curves symmetrical to those in Fig. 1a with respect to the ordinate $\eta_0 = 0.5$ will also be roots of Eqs. (22). As the point with coordinates $(\varkappa, \eta_0)$ approaches these curves, parameter $\alpha \to +\infty$. The limit approached by the shearing stress at $\alpha \to \pm\infty$ rises as $\varkappa \to 0$.

A peculiar feature here is the intersection of curves plotted for different values of n. For n = 1, curve 2 is identical to the analogous curve in [1] after correspondence between the respective systems of coordinates have been established.

The intersection points of the curves with the ordinate $\eta_0 = 0.5$ yield the values $\varkappa = \varkappa_*$, which are critical. For all $\varkappa < \varkappa_*$ there exists a steady flow mode, for all $\varkappa \geq \varkappa_*$ a steady flow mode is impossible and a loss of thermal stability occurs. As $\varkappa \to \varkappa_*$, the temperatures and the velocities rise rapidly. The critical $\varkappa_*$ is a monotonically increasing function of n and it depends neither on the temperature drop at the plates nor on the velocity of the moving plate.

Curves of $\varkappa(\eta_0)$ at various values of n are shown in Fig. 1b for the case of a thermally insulated lower plate. Here the curves are not symmetrical and, within the $0 < \eta_0 < 0.5$ range, they pass through a maximum which determines the critical $\varkappa_*$ value. The left-hand branches of these curves determine the limiting values of η_0 when $\alpha \to +\infty$, while the right-hand branches determine the limiting values of η_0 when $\alpha \to -\infty$. As n increases, the slope of these curves as well as the value of $\varkappa_*$ increase and, at the same time, the maximum of each curve approaches the ordinate $\eta_0 = 0.5$. Curve 2 is identical to curve 1 in Fig. 2 in [1].

We will now determine the ranges of parameter $\varkappa$ and α values within which the various flow modes occur. The analysis will be applied to the case of one thermally insulated plate. Steady flow is possible if $\varkappa < \varkappa_*$. Inserting $\eta_0 = 0$ and $\eta_0 = 1$ into the equation $\Delta(\varkappa, \eta_0) = 0$ yields respectively two values: $\varkappa_0$ and $\varkappa_1$, $\varkappa_0 > \varkappa_1$. If $\varkappa < \varkappa_1$, then all three flow modes are possible: $\eta_0 \leq 0$, $0 \leq \eta_0 \leq 1$, and $\eta_0 \geq 1$. At a given $\varkappa < \varkappa_1$, inserting into Eq. (14) (but with coefficients A_1 and B_1 corresponding to the case of one thermally insulated plate) $\eta_0 = 0$ and $\eta_0 = 1$ will yield respectively two values: α_0 and α_1:

$$\alpha_0 = A_1 \sum_{k=0}^{\infty} \frac{a_k}{vk+n+2} + B_1 \sum_{k=0}^{\infty} \frac{b_k}{vk+n+1},$$

$$\alpha_1 = A_1 \sum_{k=0}^{\infty} \frac{a_k}{vk+n+2} - B_1 \sum_{k=0}^{\infty} \frac{b_k}{vk+n+1}. \tag{24}$$

These flow modes will occur when, respectively, $\alpha \geq \alpha_0$, $\alpha_1 \leq \alpha \leq \alpha_0$, and $\alpha \leq \alpha_1$.

When $\varkappa_1 \leq \varkappa \leq \varkappa_0$, only two flow modes are possible: with η_0 within the gap and $\eta_0 \leq 0$; moreover, $\eta_0 \leq 0$ when $\alpha \geq \alpha_0$. At all other values of α, positive as well as negative, $0 \leq \eta_0 \leq 1$. If $\varkappa_0 \leq \varkappa < \varkappa_*$, then η_0 occurs within the gap at any value of α.

In the case of thermostaticized plates, $\varkappa_0 = \varkappa_1$ owing to the symmetry of the curves and, therefore: only one flow mode with $0 \leq \eta_0 \leq 1$ occurs when $\varkappa_0 \leq \varkappa < \varkappa_*$, but all three flow modes (with $\eta_0 \leq 0$, $0 \leq \eta_0 \leq 1$, and $\eta_0 \geq 1$) occur when, respectively, $\alpha \geq \alpha_0$, $\alpha_1 \leq \alpha \leq \alpha_0$, and $\alpha \leq \alpha_1$ if $\varkappa \leq \varkappa_0$ where α_0 and α_1 are determined from (24).

When $\alpha = 0$ (stationary plates), $0 < \eta_0 < 1$ and, therefore, always $\alpha_1 < 0$. This means that the flow mode with $\eta_0 \geq 1$ can occur only at negative values of α. It must be emphasized here that $\alpha > 0$ in screw pumps and extruders. For this reason, these devices can operate in only two modes: $\eta_0 \leq 0$ and $0 \leq \eta_0 < 1$.

The temperature profile in the gap between two plates is shown in Fig. 2 for n = 3, $\alpha = 1.34$, and $\varkappa = 1.22$. The solid curve represents the case of both thermostaticized plates at the same temperature ($\theta_0 = 1$), the dashed curve corresponds to the case where the lower plate is thermally insulated.

According to the diagram, the temperature profile between two thermostaticized plates passes through a maximum. The asymmetry of the solid curve is explained by the asymmetry of the velocity profile in a generalized Couette flow. When the lower plate is thermally insulated, then, not surprisingly, the maximum temperature is reached at the lower plate.

The relation between the dimensionless flow rate Q and the dimensionless pressure gradient $a = A/A_m$ (A_m denoting the maximum pressure gradient in an isothermal flow ($\varkappa = 0$) of a Newtonian fluid (n = 1), which occurs at a zero flow rate) has been analyzed for various values of the parameters. Parameters α and $\varkappa$ can be expressed as follows:

$$\alpha = \alpha_m / a^n, \quad \varkappa = \varkappa_m a^{\frac{n+1}{4}},$$

where

$$\alpha_m = \frac{v_0}{k_0 h (A_m h)^n}, \quad \varkappa_m^4 = \frac{k_0 \beta (A_m h)^{n+1} h^2}{\lambda J}.$$

Curves of $Q(a)$ at $\alpha_m = 0.168$ and $\varkappa_m = 2.52$ in Fig. 3 for two values of n. As n increases, the flow rate increases too and at a rate which becomes higher at higher values of a. Characteristic here is the intersection of solid and dashed lines. To each such pair of curves there corresponds a value $a = a_0$ which is the abscissa of the intersection point. When $a < a_0$, the flow rate under adiabatic conditions at the lower plate is higher than under isothermal conditions, and conversely when $a > a_0$.

The effect of the dissipation parameter $\varkappa_m$ on the $Q(a)$ relation is shown in Fig. 4 for the case of a thermally insulated lower plate. According to the diagram, this effect on the flow rate is strongest when the extruder screw operates near free flow or near shutoff. An increase in $\varkappa_m$, moreover, causes an increase in the flow rate when a is small and a decrease in the flow rate when a is sufficiently large. In the case of thermostaticized plates, these curves follow the same trend.

NOTATION

h	is the distance between the plates;
y	is the coordinate;
y_0	is the coordinate of the point where the shearing stress is zero;
v	is the velocity;
v_0	is the velocity of the upper plate;
$dp/dx = A$	is the pressure gradient;

τ	is the shearing stress;
k, n	are the rheological parameters;
λ	is the thermal conductivity of the fluid;
J	is the mechanical equivalent of heat;
T, T_1, T_2	are the temperature of the fluid, of the lower plate, and of the upper plate respectively;
Q	is the dimensionless flow rate;
$\eta = y/h$, $w = v/v_0$, $\theta = 1 + \beta(T - T_2)$	are the dimensionless coordinate, velocity, and temperature respectively;
$\alpha = v_0/k_0h(Ah)^n$, $\varkappa^4 = k_0\beta(Ah)^{n+1}h^2/\lambda J$, $\theta_0 = 1 + \beta(T_1 - T_2)$	are the dimensionless parameters;
$\varkappa_*$	is the critical value of $\varkappa$.

LITERATURE CITED

1. S. A. Bostandzhiyan and A. M. Stolin, Inzh. Fiz. Zh., 17, No. 1 (1969).
2. E. Jahnke and F. Emde, Tables of Functions with Formulas and Curves [Russian translation], Fizmatgiz, Moscow (1959).
3. S. A. Bostandzhiyan and A. M. Stolin, Izv. Akad. Nauk SSSR, Mekhanika, 1, 185 (1965).